$$y = 5x^{2/3} - x^{5/3}$$

$$y' = \frac{10}{3}x^{-\frac{1}{3}} - \frac{5}{3}x^{2/3}$$

$$\overline{x^{-\frac{1}{3}}} \qquad \overline{x^{-\frac{1}{3}}}$$

$$= x^{-\frac{1}{3}}\left(\frac{10}{3} - \frac{5}{3}x\right)$$

$$\frac{1}{\sqrt[3]{x}}$$

$y' = 0$ when $x = 0, 2$ local extrema

undef. zero

$$y'' = x^{-\frac{1}{3}}\left(-\frac{5}{3}\right) + \left(\frac{10}{3} - \frac{5}{3}x\right)\left(-\frac{1}{3}x^{-\frac{4}{3}}\right)$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} = \frac{-20}{9}$$ concave down

$$y'' = -\frac{10}{9}x^{-4/3} - \frac{10}{9}x^{-\frac{1}{3}}$$

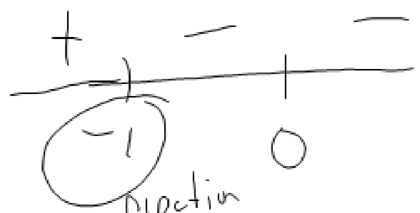
$$\overline{x^{-\frac{1}{3}}}$$

$$y'' = -\frac{10}{9}x^{-\frac{1}{3}}\left(x^{-1} + 1\right)$$

$$y'' = \frac{-10}{9x^{\frac{1}{3}}}\left(\frac{1}{x} + 1\right)$$

zero $= -1$

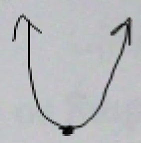
undef. $= 0$



inflection pt.

Main Ideas

- Critical Points (values)
  - where $f'(x)$ is zero or undefined ((if $f(x)$ exists at that point))

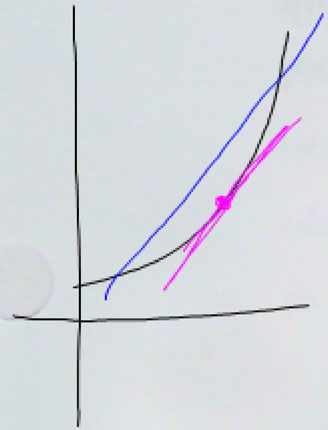


- Extrema
  - Absolute min/max → ~~only~~ usually on closed interval
  - relative (local) min/max → on an open interval
  - Can occur when $f'(x)=0$, $f'(x)=$ undef., or at endpoints
  - Sign change needed (1st derivative Test $\underset{0}{-}\;\underset{2}{-}\;+$)

- Increasing/decreasing
  - If $f'(x)>0$ at each point of (a,b), then $f(x)$ increases on $[a,b]$
  - If $f'(x)<0$ at each point of (a,b), then $f(x)$ decreases on $[a,b]$
  - If $f'(x)=0$ at each point of (a,b), then $f(x)$ is constant on $[a,b]$

- Increasing / decreasing
  - If $f'(x) > 0$ at each point of (a,b), then $f(x)$ increases on $[a,b]$
  - If $f'(x) < 0$ at each point of (a,b), then $f(x)$ decreases on $[a,b]$
  - If $f'(x) = 0$ at each point of (a,b), then $f(x)$ is constant on $[a,b]$

- Mean Value Thrm
  - If $f(x)$ is continuous at every point of the closed interval $[a,b]$ and differentiable at every interior point on (a,b), then there is at least one point c in (a,b) such that

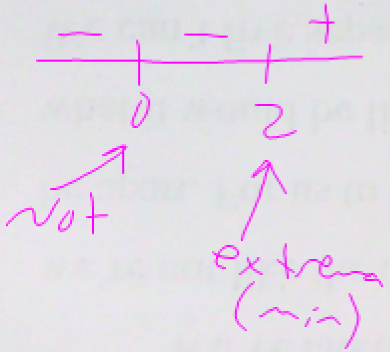


$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\underset{\text{slope tangent}}{\downarrow} \quad = \quad \underset{\text{slope secant}}{\downarrow}$$

Main Ideas cont.

- First derivative test
  - If c is a critical number of the function $f(x)$
    1. If $f'(x)$ changes from neg. to pos. at c, then $f(x)$ has a relative min at c.
    2. If $f'(x)$ changes from pos. to neg. at c, then $f(x)$ has a relative max at c.



- Concavity
  - If $f''(x)$ is positive, then $f(x)$ concave up
  - If $f''(x)$ is neg., then $f(x)$ concave down

- Point of Inflection
  - point where $f(x)$ has a tangent line and where the

- Point of Inflection
  - A point where $f(x)$ has a tangent line and where the concavity changes

- Second derivative test
  - If $f'(c)=0$ and $f''(c)<0$, then $f(x)$ has a local max at c
  - If $f'(c)=0$ and $f''(c)>0$, then $f(x)$ has a local min at c
  - If $f''(c)=0$, the test fails

Do these problems analytically

① Find the extrema of $f(x) = 2x - 3x^{2/3}$ on interval $[-1, 3]$

② Find the extrema of $f(x) = 2\sin x - \cos 2x$ on interval $[0, 2\pi)$

③ Find points of inflection, extrema, and discuss concavity of the graph of $f(x) = x^4 - 4x^3$

④ Show how to sketch a "good" graph of $y = x^4 - 12x^3 + 48x^2 - 64x$ analytically

⑤ Analyze graph of $f(x) = \dfrac{\cos x}{1 + \sin x}$.

do prep book – concavity pp. 128 – 133

③ $f'(x) = 4x^3 - 12x^2$

$0 = 4x^2(x-3)$

$x = 0, 3$

$-$ $-$ $+$

0 3

min

$f''(x) = 12x^2 - 24x = 12x(x-2)$

$0 = 12x(x-2)$

$x = 0, 2$

$+$ $-$ $+$

0 2

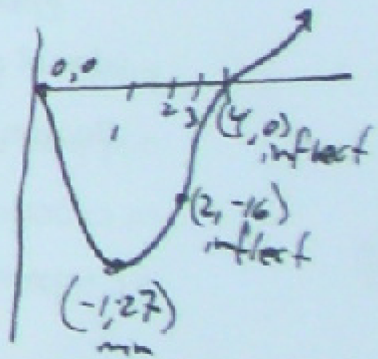
inflect inflect.

④ $f(x) = x(x-4)^3$ $f'(x) = 4(x-1)(x-4)^2$ $f''(x) = 12(x-4)(x-2)$

x-int: $(0,0)(4,0)$, y-int: $(0,0)$, end behavior ∞ both sides, critical #'s $x=1$, $x=4$

min

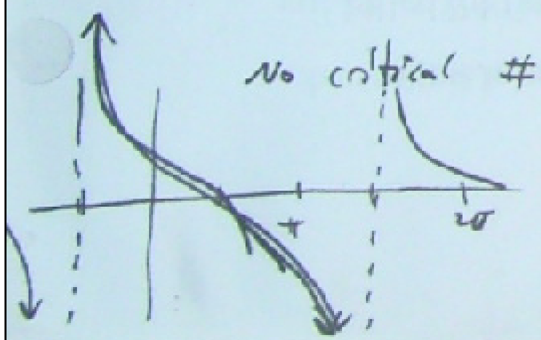
inflect. pts $x = 2, 4$



⑤ $f'(x) = -\dfrac{1}{1+\sin x}$ $f''(x) = \dfrac{\cos x}{(1+\sin x)^2}$

x-int: $(\frac{\pi}{2}, 0)$ y-int $(0,1)$ vert. asymp. $x = -\frac{\pi}{2}, \frac{3\pi}{2}$

No critical #'s inflect. point $x = \frac{\pi}{2}$



Answers

12/14/09
cont.

① $f'(x) = 2 - \frac{2}{x^{\frac{1}{3}}} = 2\left(\frac{x^{\frac{1}{3}} - 1}{x^{\frac{1}{3}}}\right)$

Zeros at $x = 1$
undef at $x = 0$

x	-1	0	1	3
$f(x)$	-5	0	-1	-0.24
	min (abs)	max (abs)	local min	local max

$+ \quad 0 \quad - \quad 1 \quad +$

0: max, 1: min

② $f'(x) = 2\cos x + 2\sin 2x = 0$

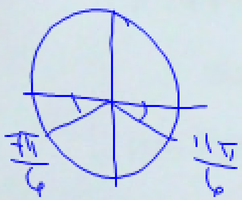
$2\cos x + 4\cos x \sin x = 0$

$(2\cos x)(1 + 2\sin x) = 0$

$\frac{\pi}{2}, \frac{3\pi}{2} \qquad \frac{7\pi}{6}, \frac{11\pi}{6}$

$1 + 2\sin x = 0$

$\sin x = -\frac{1}{2}$



$+ \quad - \quad + \quad - \quad +$

0	$\frac{\pi}{2}$	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
min	max (abs)	min (abs)	max	min (abs)	max

③ $4x^3 - 12x^2 \qquad f''(x) = 12x^2 - 24x = 12x(x-2)$

$0 = 12x(x-2)$